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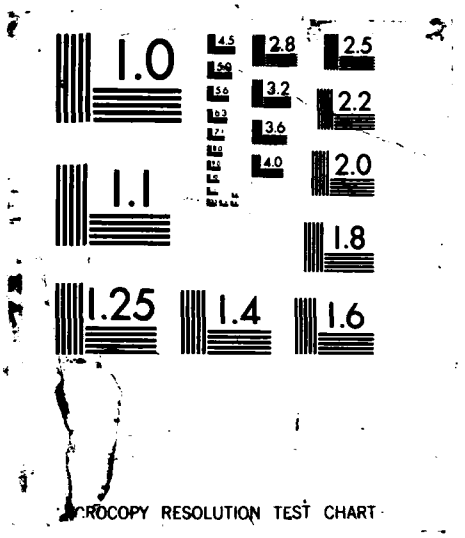
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A Parametric Patch Surface Geometry Definition for the Three-Dimensional Retarded Potential Technique

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A parabolically parametric surface patch basis has been developed for the geometric calculations of the discretized retarded potential integration. Arbitrary three-dimensional surface geometries can be handled, with a present restriction on the allowed orientation of the surface patch grid lines. A test problem confirms the accuracy of the formulation.

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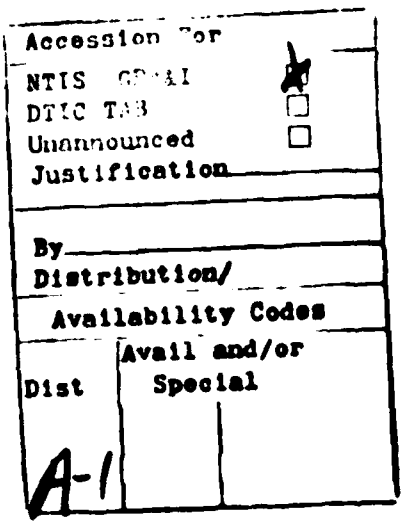

A PARAMETRIC PATCH SURFACE GEOMETRY DEFINITION FOR THE THREE-DIMENSIONAL RETARDED POTENTIAL TECHNIQUE

INTRODUCTION

This study is a part of the effort to apply the retarded potential integral to three-dimensional problems in fluid-structure interaction for arbitrary bounding geometries. This paper will be confined to a discussion of the geometric aspects of retarded potential computations. A related paper covers the application of the three dimensional technique to problems of shock [6].

Mitzner [1,2] developed a method for discretizing the integral spatially and temporally, and applied the technique to the problem of a spherical rigid submerged body with two-dimensional axisymmetry. Huang, Everstine, and Wang [3] extended the method to handle a non-rigid boundary condition, and tested the case of a spherical elastic shell, modeled axisymmetrically.

This work builds upon the computer program developed by Mitzner and extended to non-rigid boundaries by Huang, Everstine, and Wang. A fully three- dimensional boundary element formulation for arbitrarily shaped surfaces has been implemented. The entire approach is designed to be compatible with finite element methods, important in the case of coupled problems involving non-rigid submerged structures.

In the new development, the surface geometry is totally defined by the boundary element input data. The Cartesian coordinates, curvatures, and surface coordinate base vectors, which are needed at various points by the coefficients calculation routine, are all derived from the natural surface coordinate interpolation functions of the boundary patch. The numerical definition of geometry is required for a problem-independent retarded potential analysis capability, particularly when coupled to a finite element program. This avoids the need to individually tailor surface parametric equations for each problem geometry encountered, which would require reprogramming. Beyond this consideration, for an arbitrary geometry it may be difficult to obtain a set of closed-form parametric equations to define a particular problem.

RETARDED POTENTIAL FORMULATION

A discussion of the retarded potential equation and its discretized approximation may be found in Refs. [1,2,3] with a derivation in Ref.[4]. The form of the equation for calculation of total pressures, p, on the boundary of a smooth, closed surface in an infinite fluid is as follows:

$$p(\bar{x},t) = 2p^{inc}(\bar{x},t) - \rho/2\pi \int_S \frac{\ddot{w}(\bar{x}',t')}{R}\, dS'$$

$$+1/2\pi \int_S \left\{ p(\bar{x}',t') + \frac{R}{c}\frac{\partial p}{\partial t'}(\bar{x}',t') \right\} \frac{1}{R^2}\frac{\partial R}{\partial n'}\, dS' \qquad (1)$$

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where ρ and c are the fluid density and sonic velocity, respectively, $\bar{x}$ is the surface position of the field point, $\bar{x}'$ is the surface position of an integration point, $R = |\bar{x} - \bar{x}'|$ is the distance between field and integration points, $t' = t - \frac{R}{c}$ is the retarted time of the integration point, p^{inc} is the incident pressure field in the fluid, n' is the surface normal at the integration point directed out of the fluid, and $\ddot{w}'$ is the normal direction acceleration of the surface.

From the integral, the basic geometric influence factors to be evaluated are as follows:

$$R = |\bar{x} - \bar{x}'| \tag{3a}$$

$$dS'/R \tag{3b}$$

$$\text{and} \quad \frac{1}{R^2} \frac{\partial R}{\partial n'} dS' \tag{3c}$$

Computation of the discretized forms of these for the parametric surface patch is the subject of this report.

DISCRETIZATION

For computation, the retarded potential integral is discretized into a set of algebraic equations. The surface pressure field is approximated by subdividing the surface into zones of spatially constant pressure. The surface normal acceleration field, if not identically zero, is also approximated as constant on the surface of the same zone. Once the pressure and acceleration (and pressure derivative) are approximated as constant over a part of the surface, these quantities can be factored out of the integral expressions. This results in integrals which are dependent on surface geometry alone. There is a need to further subdivide the surface of zones into a mesh of subzone elements in order to accurately numerically integrate the geometric influence factors, as well as to accurately obtain the time-delayed influences of pressures and accelerations from each preceding time step [1,2,3]. Then, the acceleration influence and pressure influence integrals over the subzone element, S_e, are as follows:

$$\int_{S_e} \frac{dS'}{R} \tag{4a}$$

$$\int_{S_e} \frac{1}{R^2} \frac{\partial R}{\partial n'} dS' \tag{4b}$$

Discussion of the discretization of retarded time into steps is left to the related report on the application of the technique to shock response problems [6] or can be found in the original references [1,2,3].

In the original solution procedure, the computations are based on a surface coordinate system, (α,β), common to the entire boundary and, unique to the to the particular geometry. Then the subdivision of the surface into zones and subzone elements is made to follow the lines of the coordinate system [1,2]. The flexibility of the new development allows for a series of independent surface coordinate systems, one per surface patch, as will be described later. In the following discussion the dimensions of a subzone element, expressed in surface coordinates, are $\Delta\alpha$ by $\Delta\beta$, and the element's surface area is S_e.

Computationally, the subzone elements are classified into two types, singular and non-singular. The area of a non-singular element is approximated as a flat surface tangent to the integration point at its center. With the boundaries of the element following the lines of the surface coordinate system, the cross product of the Euclidean base vectors becomes the key quantity in obtaining the physical size of the element and its normal-direction vector. The resulting expressions for $\int_{S_e} dS'/R$ and $\int_{S_e} 1/R^2 \, \partial R/\partial n' \, dS'$ over the non-singular element are

$$\Delta\alpha\ \Delta\beta\ |\overline{g}_\alpha \times \overline{g}_\beta|/R \quad [3a] \tag{5a}$$

and

$$\Delta\alpha\ \Delta\beta/R^3\ (\overline{x}'-\overline{x}) \cdot (\overline{g}_\alpha \times \overline{g}_\beta) \quad [1] \tag{5b}$$

respectively. Distance measurement between a non-singular element and the field point of a zone is approximated by locating the field point at the center of the field zone and the integration point at the center of the integration subzone element.

For the surface area immediately surrounding the field point, modified calculations must be used to compute the pressure and acceleration integration influences, because of the close proximity of the field point and the presence of a singularity at the field point itself. For the pressure integral, the surface surrounding the field point is approximated by an osculating paraboloid, with principal curvatures matching at the central field point, and an exact integral is evaluated over this surface. In the current implementation, this integral has been simplified by requiring the surface coordinate system to be aligned with the principal curvature directions. With the simplification, the expression for $\int_{S_e} 1/R^2\ \partial R/\partial n'\ dS'$ over the singular element becomes

$$2(\kappa_\alpha Y_o \ln\frac{r_o+X_o}{Y_o} + \kappa_\beta X_o \ln\frac{r_o+Y_o}{X_o}) \quad [1] \tag{6}$$

where

$$X_o = 1/2\ |\overline{g}_\alpha|\ \Delta\alpha$$

$$Y_o = 1/2\ |\overline{g}_\beta|\ \Delta\beta$$

and

$$r_o = (X_o^2+Y_o^2)^{1/2}$$

In the above formulas the κ's are the principal curvatures at the field zone center, with the rest of the quantities derived from the Euclidean base vectors and the element size, as measured in surface coordinates. The singular element computations require the distance, R, in the expression R/c of the pressure derivative term. In this case the integration point is taken to be a distance of one-fourth the diagonal dimension of the singular subzone from the zone-centered field point ($\frac{1}{2}r_0$).

For the acceleration influence, a cruder approximate integral is used here. This is acceptable because the weaker singularity yields a less significant influence of the near-by accelerations. The approximation involves replacing the rectangular element with a circle of the same area, and integrating the expression dS'/R over a flat surface. The result is $4(\pi X_o Y_o)^{1/2}$ [3a]. As can be seen from the above expressions, the influence coefficients calculations require the evaluation of three basic local geometric quantities: the position vector, the coordinate base vectors, and the curvatures. The subzone element sizes are determined from the user—specified subzone element mesh refinement.

PARAMETRIC PATCH SURFACE DEFINITION

The surface geometry definition implemented for the retarded potential method is the same as that used for isoparametric finite element shells in three dimensional space [5]. The boundary surface is divided into patches to coincide with zones of constant pressure. When coupled to a structural analysis code ,these also coincide with the shell surface finite elements, allowing one surface definition to be given to both the structural problem and the fluid pressure problem. Each surface patch has its own curvilinear, non-dimensional coordinate system, which follows the surface contour with coordinate values varying from -1 to +1. This is referred to as a natural coordinate system, with coordinate directions r and s. The surface zone used in this study has a 3 × 3 arrangement of nodal points to match the 9-noded version of the ADINA shell finite element. The nodes lie at whole number values of the parametric coordinate pairs, designated (r_i,s_i) for node i. The nodal coordinates, $\overline{X}_i^N$, define the Cartesian coordinates of the entire element surface through biquadratic interpolation functions of the two natural coordinates, as shown in Eq. (7)

$$\bar{X}(r,s) = \sum_i H_i(r,s)\ \bar{X}_i^N \tag{7}$$

The interpolation functions are built up from elementary terms of the form, F, shown as Eq. (8), which, in turn, are products of the unidirectional factors, Q, shown in Eqs. (8a).

$$F(r,s,r_i,s_i) = Q(r,r_i)\ Q(s,s_i) \tag{8}$$

$$\text{for } |r_i|=1 \qquad Q(r,r_i) = \tfrac{1}{2}\ (1+r_i r)$$

$$\text{for } |r_i|=0 \qquad Q(r,r_i) = \ (1-r^2) \tag{8a}$$

The function, F, has a value of one at the node i and a value of zero at all edges of the finite element that do not touch node i. For edge and corner nodes, achieving an interpolation function that has a zero value at all nodes other than i requires subtraction of fractional values of other interpolation functions from the F expression. The full interpolation functions are related to the F expressions and to each other by the following expressions.

$$\begin{aligned}
H_1 &= F(r,s,\ 1,\ 1) - .5\ (H_5+H_8) - .25\ H_9 \\
H_2 &= F(r,s,-1,\ 1) - .5\ (H_5+H_6) - .25\ H_9 \\
H_3 &= F(r,s,-1,-1) - .5\ (H_6+H_7) - .25\ H_9 \\
H_4 &= F(r,s,\ 1,-1) - .5\ (H_7+H_8) - .25\ H_9 \\
H_5 &= F(r,s,\ 0,\ 1) \qquad\qquad - .5\ H_9 \\
H_6 &= F(r,s,-1,\ 0) \qquad\qquad - .5\ H_9 \\
H_7 &= F(r,s,\ 0,-1) \qquad\qquad - .5\ H_9 \\
H_8 &= F(r,s,\ 1,\ 0) \qquad\qquad - .5\ H_9 \\
H_9 &= F(r,s,\ 1,-1)
\end{aligned} \tag{9}$$

where $H_i = H_i(r,s)$.

GEOMETRIC COMPUTATIONS

As discussed earlier, three types of geometric information are needed for the calculation of the retarded potential influence coefficients.

1. The Cartesian Position Vectors are needed to locate the field and integration points in space.

2. Base Vectors (which the derivatives of the position vector with respect to the natural coordinate directions) are needed to determine the orientation and size of the integration subzones.

3. Principal Surface Curvatures are required at the field points to determine the osculating paraboloid for the singular subzone calculation discussed earlier.

The expressions for the base vectors, $\bar{g}_r$ and $\bar{g}_s$, can be obtained by taking the r- and s-direction derivatives (i.e. a derivative with respect to one of the natural coordinates) of the position vector interpolation function and are given by:

$$\bar{g}_q(r,s) = \frac{\partial \bar{X}}{\partial q}(r,s) = \sum_i \frac{\partial H_i}{\partial q}(r,s)\ \bar{X}_i^N \tag{10}$$

where q is r or s.

The rate of change in surface orientation is expressed as the rate of change of a unit normal vector with respect to a position change in a given direction along the surface. Because the unit vector can only change in direction and not length, its change from one point to another is a measure of angle change, with the added feature of showing the direction of the angle change. The component of the angle change in the direction of position change gives the angle of deflection. If one considers a *rate* of change in the unit normal instead, then the component in the direction of position change becomes the curvature. This component is obtained (in Eq. (11)) by taking the dot product of the rate of change of the unit vector and the unit vector in the direction of travel. [7]

$$\kappa_q = \frac{\partial \bar{i}_n}{\partial q} \frac{dq}{ds_q} \cdot \bar{i}_q \tag{11}$$

where q = either one of the natural coordinate directions, r or s

s_q = physical distance in the q direction

$\bar{i}_n$ = unit vector normal to the surface

$\bar{i}_q$ = unit vector in the q direction

The derivation of the factors in Eq. (11) are as follows:

$$\frac{dq}{ds_q} = \frac{1}{|\bar{g}_q|} \quad \text{where} \quad \bar{g}_q = \text{the base vector in the } q \text{ direction}$$

$$\bar{i}_q = \bar{g}_q / |\bar{g}_q|$$

$$\bar{i}_n = \bar{v}_n / |\bar{v}_n| \qquad \text{where} \quad \bar{v}_n = \bar{g}_r \times \bar{g}_s$$

$$\frac{\partial \bar{i}_n}{\partial q} = \frac{\partial \bar{v}_n}{\partial q} / |\bar{v}_n| - \bar{v}_n \left(\frac{\partial \bar{v}_n}{\partial q} \cdot \bar{v}_n \right) / |\bar{v}_n|^3$$

where

$$\frac{\partial \bar{v}_n}{\partial q} = \frac{\partial \bar{g}_r}{\partial q} \times \bar{g}_s - \frac{\partial \bar{g}_s}{\partial q} \times \bar{g}_r$$

As can be seen, the curvature calculation requires the evaluation of the the base vector derivatives, i.e. the second derivatives of the position vector with respect to the surface coordinates.

All the calculations for position vector, base vectors, and curvatures can use the same interpolation scheme expressed by Equations (5). By simply taking the appropriate derivatives of the F terms, the corresponding appropriate derivatives of the H functions are obtained. These, in turn, make up the expressions for all the quantities needed.

TEST PROBLEM

The retarded potential technique was studied for the rigid scattering problem to verify the 3-dimensional integration method. The test was run to duplicate the Gaussian pulse loading on a sphere case that Mitzner [1] tested. The results obtained, plotted in Fig. 3, are virtually identical to the separation of variables solution, validating the numerical geometry formulation.

The structural model used for the rigid body test problem is shown in Fig. 1. This is a quarter section of a sphere used to model an entire spherical rigid body with symmetry conditions imposed on the two cutting planes. The retarded potential program allows for such symmetry conditions. The symmetries are imposed to save computation cost and can be allowed because both the test structure and the test loadings are axisymmetric with respect to the loading direction. The subdivision into pressure

zones is more refined in the direction of wave travel in order to capture the detailed response expected in that direction. Subzone division for this problem is two refinements, one for the influence between two different pressure zones and one for the influence of a pressure zone on itself. These are shown in Figs. 2.

The computed curvatures at the centers of the pressure zones are as follows:

θ	κ_ϕ	κ_θ
9.0	0.01027	0.010043
22.5	0.01040	0.010015
31.5	0.01040	0.010015
40.5	0.01040	0.010015
49.5	0.01040	0.010015
58.5	0.01040	0.010016
67.5	0.01040	0.010015
76.5	0.01040	0.010016
85.5	0.01040	0.010015

The list of κ_ϕ shows that, even with parametric surfaces stretched over 45 degrees of circumference, the curvature values are reasonably close to the exact value, 0.01, for a sphere of radius = 100., and the values of κ_θ are virtually exact.

CONCLUSIONS

This study has developed a parametric patch formulation for the surface geometric computations of the discretized retarded potential integral. The parametric patch definition establishes the retarded potential integral as a fully developed boundary element formulation. With the added programming enhancement to eliminate the requirement that surface patch grid lines follow the principal curvature directions on the surface, complete flexibility in surface geometry specification will be achieved.

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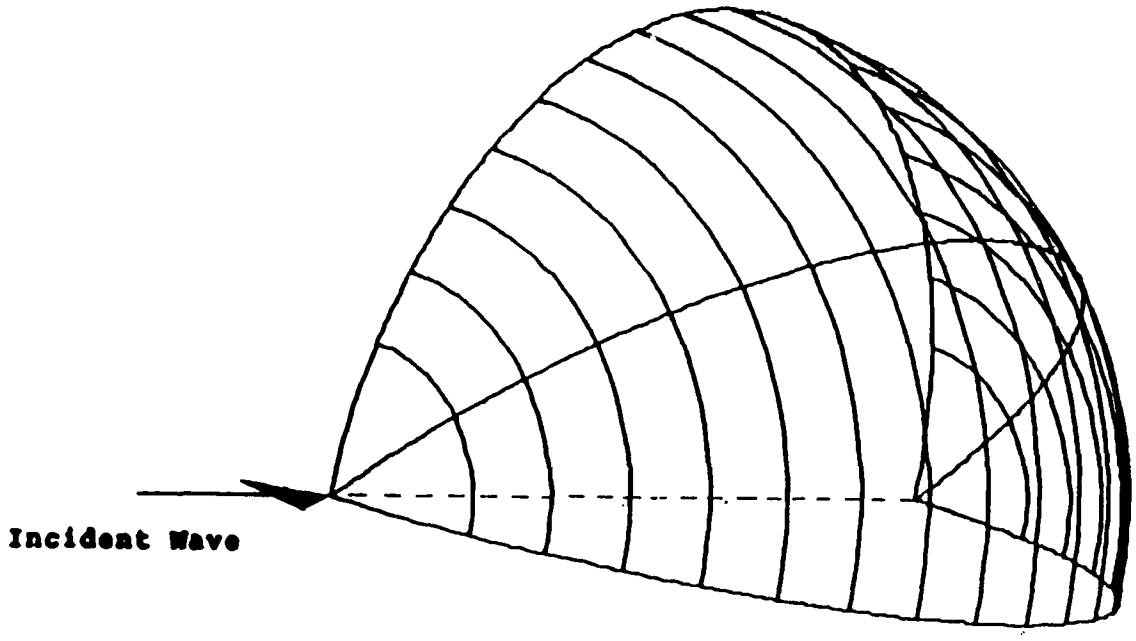


Fig. 1 – MESH of PRESSURE ZONES/FINITE ELEMENTS

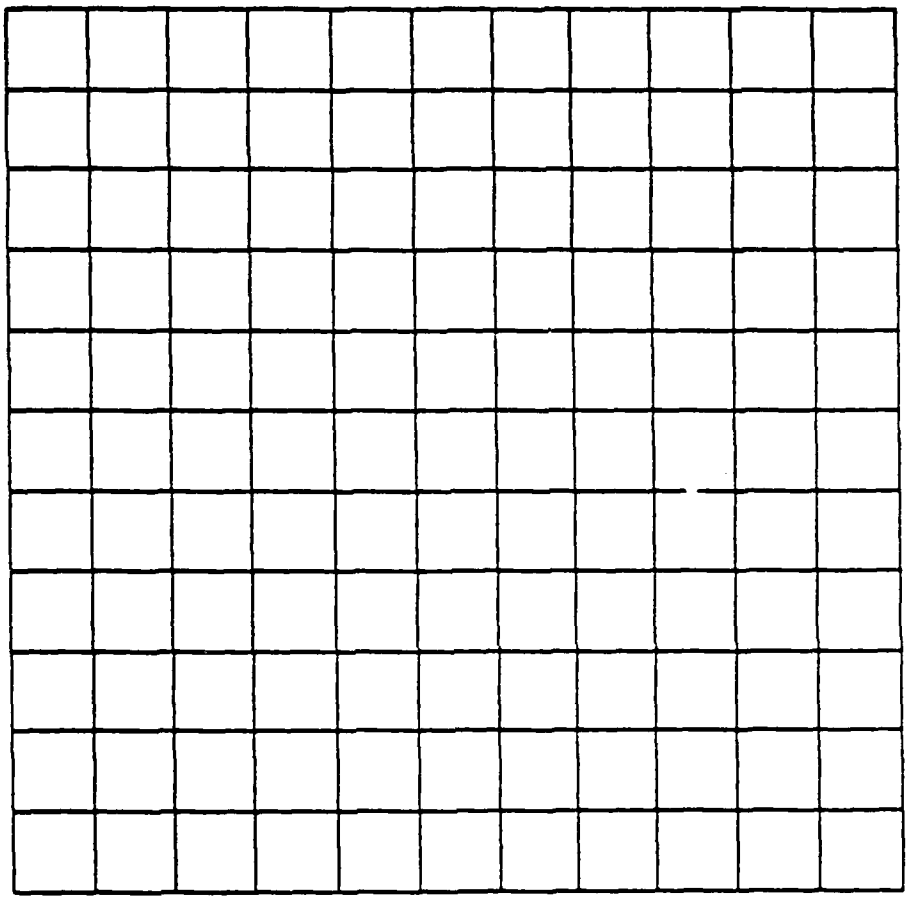


Fig. 2a — NON-SINGULAR ZONE MESH

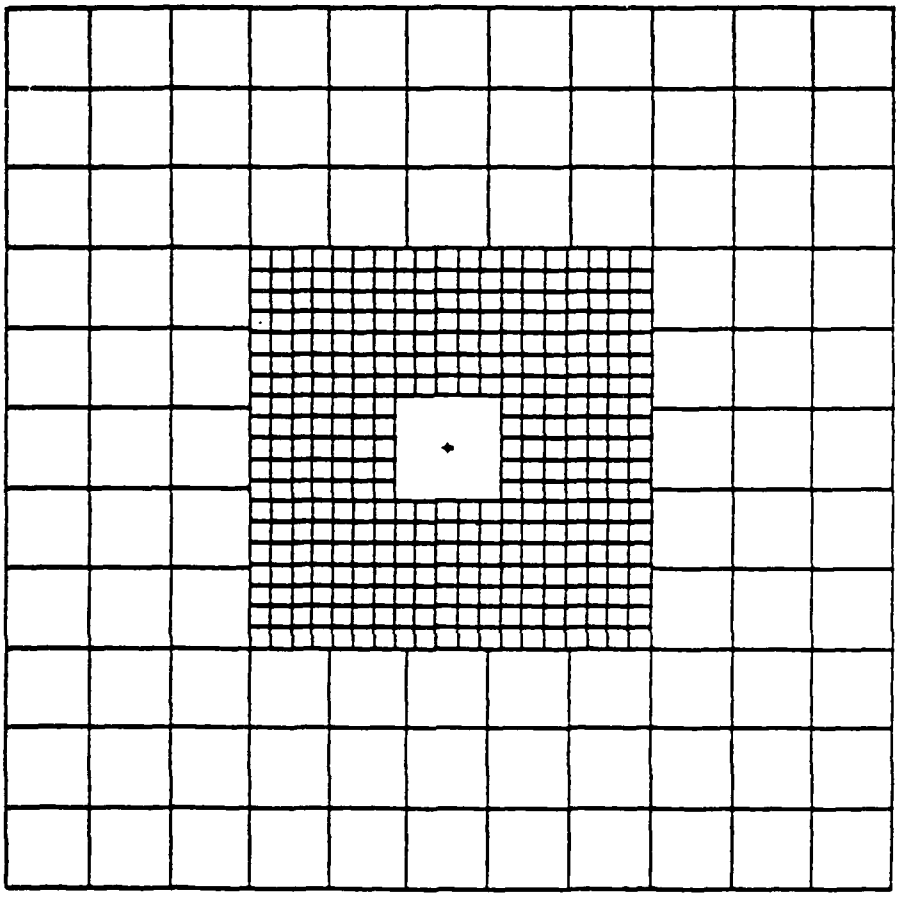
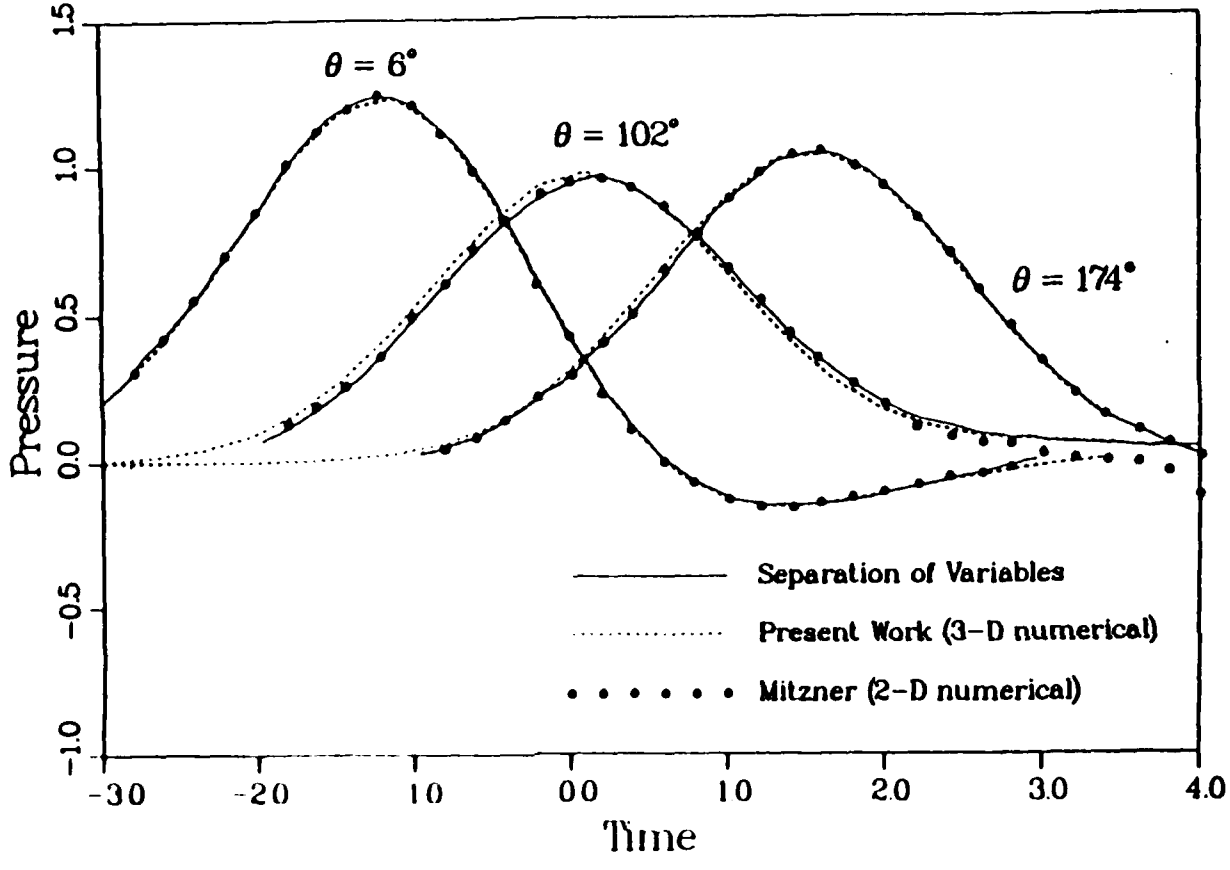


Fig. 2b — SINGULAR ZONE MESH

GAUSSIAN PULSE SCATTERING



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